General Trajectory Prior for Non-Rigid Reconstruction

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Problem

Known cameras, known 2D projections.

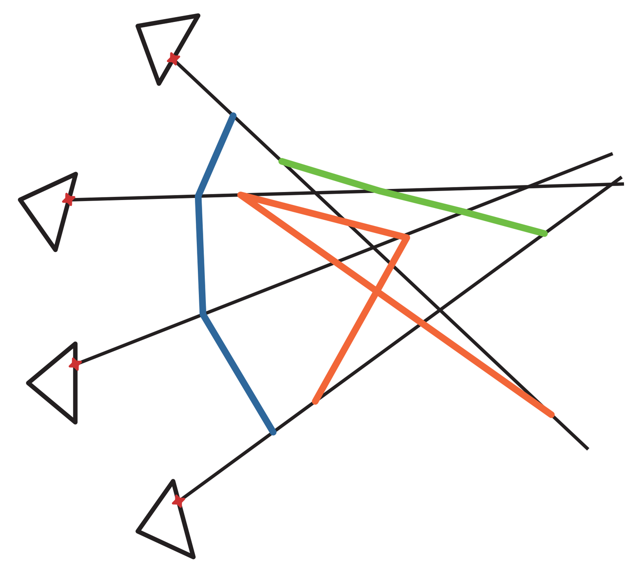
Find 3D trajectory $\mathbf{x}_1, \ldots, \mathbf{x}_F \in \mathcal{R}^3$.

Each projection defines a 2×3 linear system

$$\begin{bmatrix} \mathbf{R}_1 & & \\ & \ddots & \\ & & \mathbf{R}_F \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_F \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_F \end{bmatrix} \qquad \mathbf{R}\mathbf{x} = \mathbf{w}$$

Even with known cameras

$$2F \text{ equations} < 3F \text{ unknowns}$$



Trajectory Basis

Bregler et al. [1] used low-rank shape basis for NRSfM.

Akhter et al. [2] proposed shape-agnostic trajectory basis.

$$\mathbf{x} = \boldsymbol{\Theta}\boldsymbol{\beta}, \qquad \boldsymbol{\Theta} = \begin{bmatrix} \cdots \end{bmatrix} \otimes \mathbf{I}_3 \quad (3F \times 3K)$$

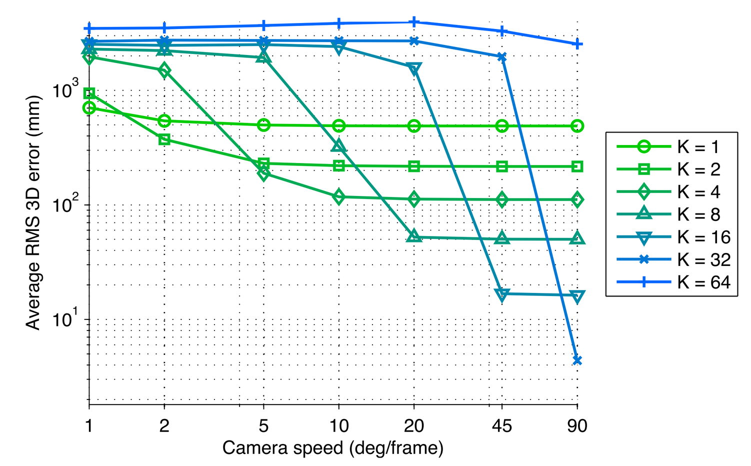
Structure obtained uniquely if $2F \geq 3K$ (or is it…)

$$\mathbf{x}^* = \boldsymbol{\Theta}\boldsymbol{\beta}^*, \qquad \boldsymbol{\beta}^* = \arg\min_{\boldsymbol{\beta}} \|\mathbf{w} - \mathbf{R}\boldsymbol{\Theta}\boldsymbol{\beta}\|_2$$

Reconstructability

Park et al. [3] noted fast camera motion led to better reconstructions.

Basis size K depends on camera speed and point trajectory.



They defined reconstructability in terms of camera trajectory $\mathbf{c}$

$$\eta(\mathbf{x}, \mathbf{c}, \boldsymbol{\Theta}) = \frac{\|(\mathbf{I} - \boldsymbol{\Theta}\boldsymbol{\Theta}^T)\mathbf{c}\|_2}{\|(\mathbf{I} - \boldsymbol{\Theta}\boldsymbol{\Theta}^T)\mathbf{x}\|_2} = \frac{\text{[camera motion orth. to basis]}}{\text{[point motion orth. to basis]}}$$

"As $\eta \to \infty$, the solution approaches the true trajectory."

Necessary *but not sufficient* for exact solution.

General Trajectory Prior

Minimise component orthogonal to basis $\mathbf{M} = (\mathbf{I} - \boldsymbol{\Theta}\boldsymbol{\Theta}^T)$

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\mathbf{M}} \quad \text{subject to } \mathbf{R}\mathbf{x} = \mathbf{w}$$

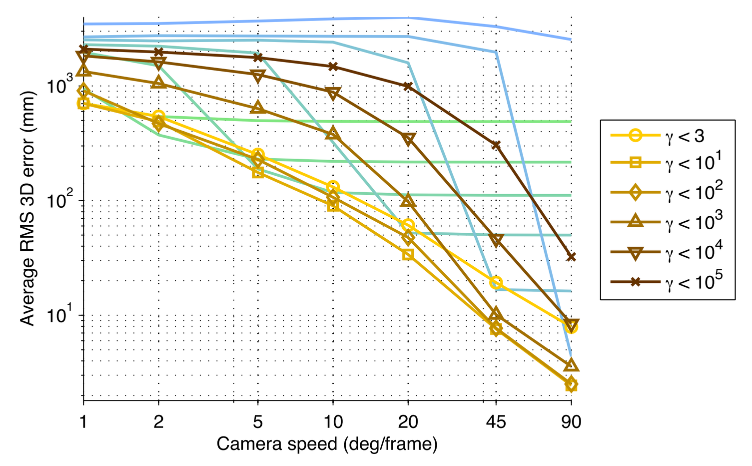
Low reconstructability results from poorly conditioned system.

Define new measure which theoretically bounds solution

$$\|\mathbf{x} - \mathbf{x}^*\|_2 \leq v(\mathbf{x}, \mathbf{R}, \mathbf{M})$$

$$v(\mathbf{x}, \mathbf{R}, \mathbf{M}) = \underbrace{\operatorname{cond}(\mathbf{R}_\perp^T \mathbf{M} \mathbf{R}_\perp)}_{\text{gain}} \underbrace{\frac{\|\mathbf{R}_\perp^T \mathbf{M} \mathbf{x}\|_2}{\|\mathbf{R}_\perp^T \mathbf{M} \mathbf{R}_\perp\|_2}}_{\text{contradiction}}$$

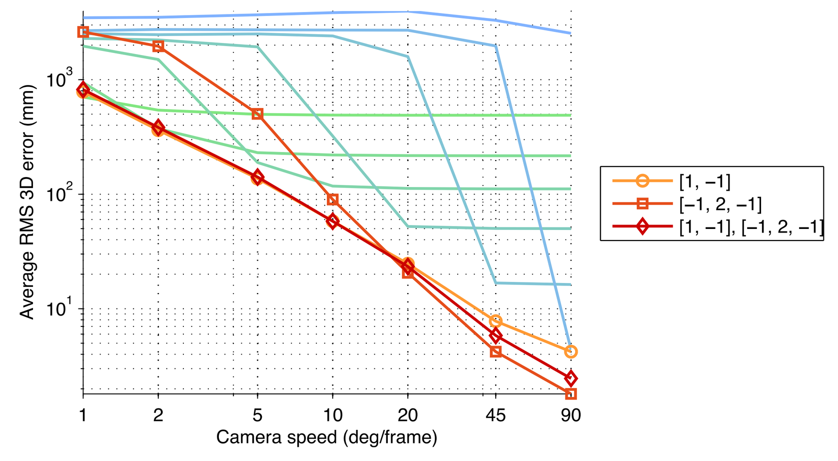
K could be chosen by limiting the "gain" (condition).



Trajectory Filters

Convolution is linear. Penalising high-pass filter response eliminates K.

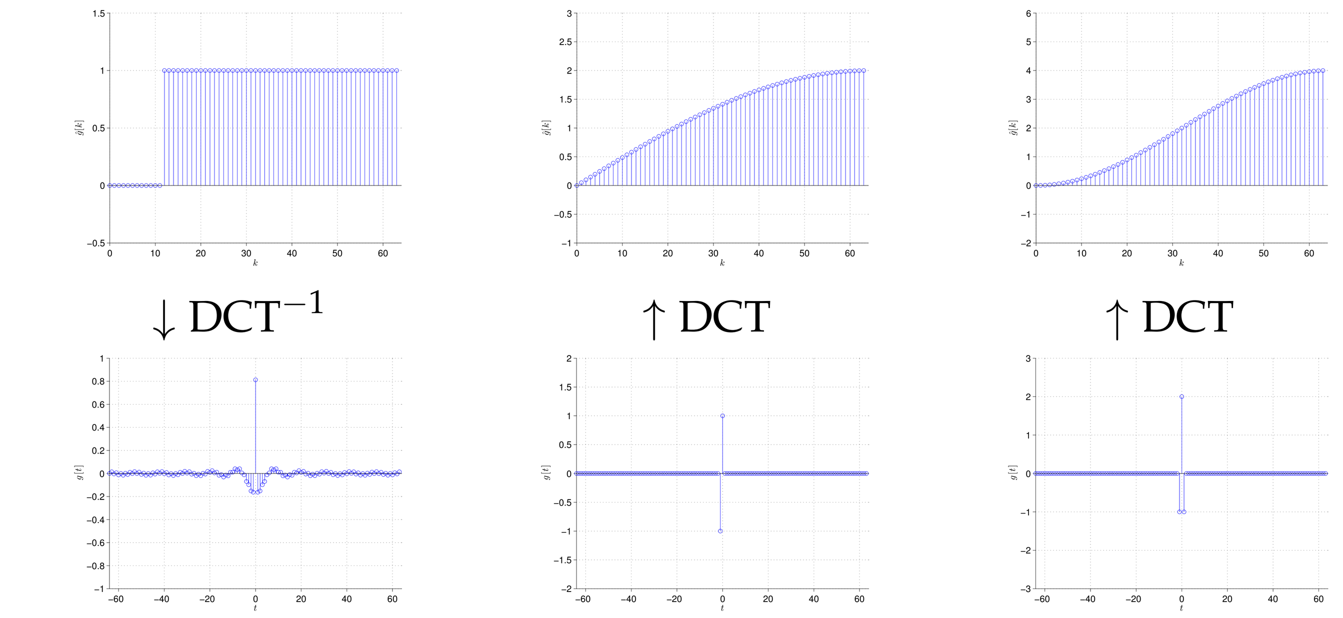
$$\|\mathbf{g} * \mathbf{x}\|_2 = \|\mathbf{x}\|_{\mathbf{M}} \quad \text{with } \mathbf{M} = \mathbf{G}^T\mathbf{G}$$



Convolution-DCT Duality

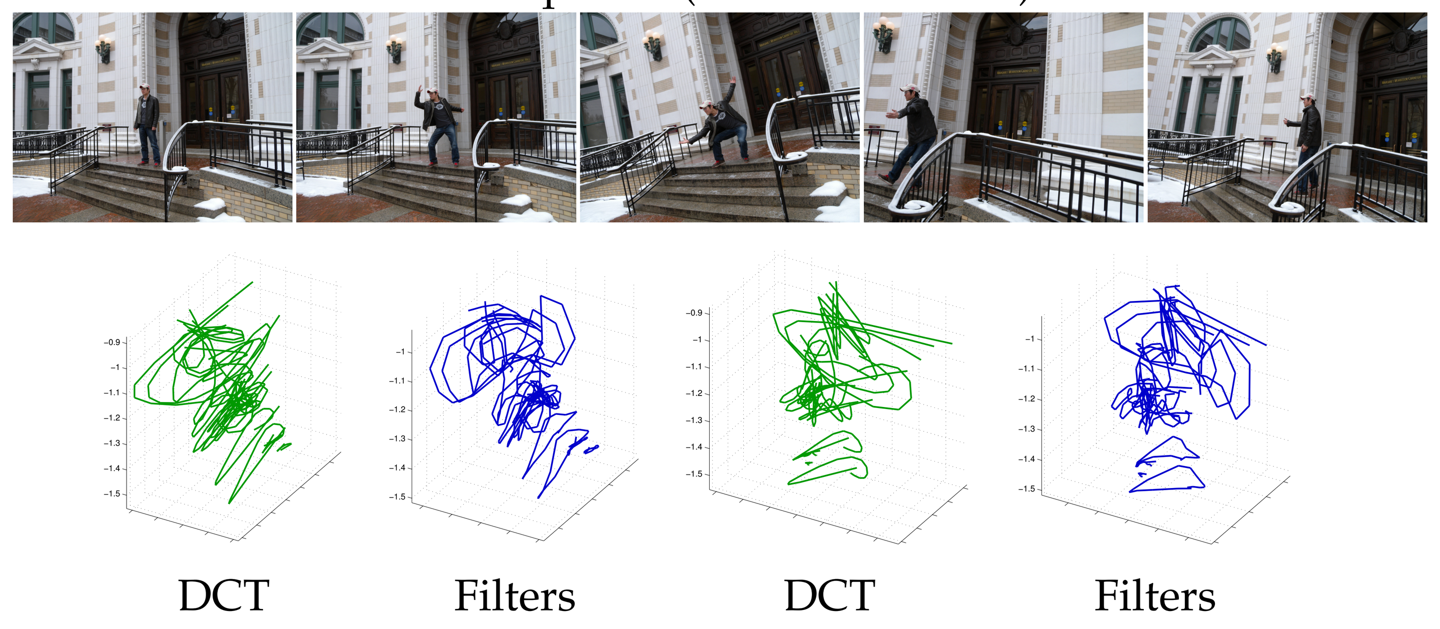
Eigenvectors of symmetric convolution form the DCT basis.

Priors with more non-zero eigenvalues are better constrained.

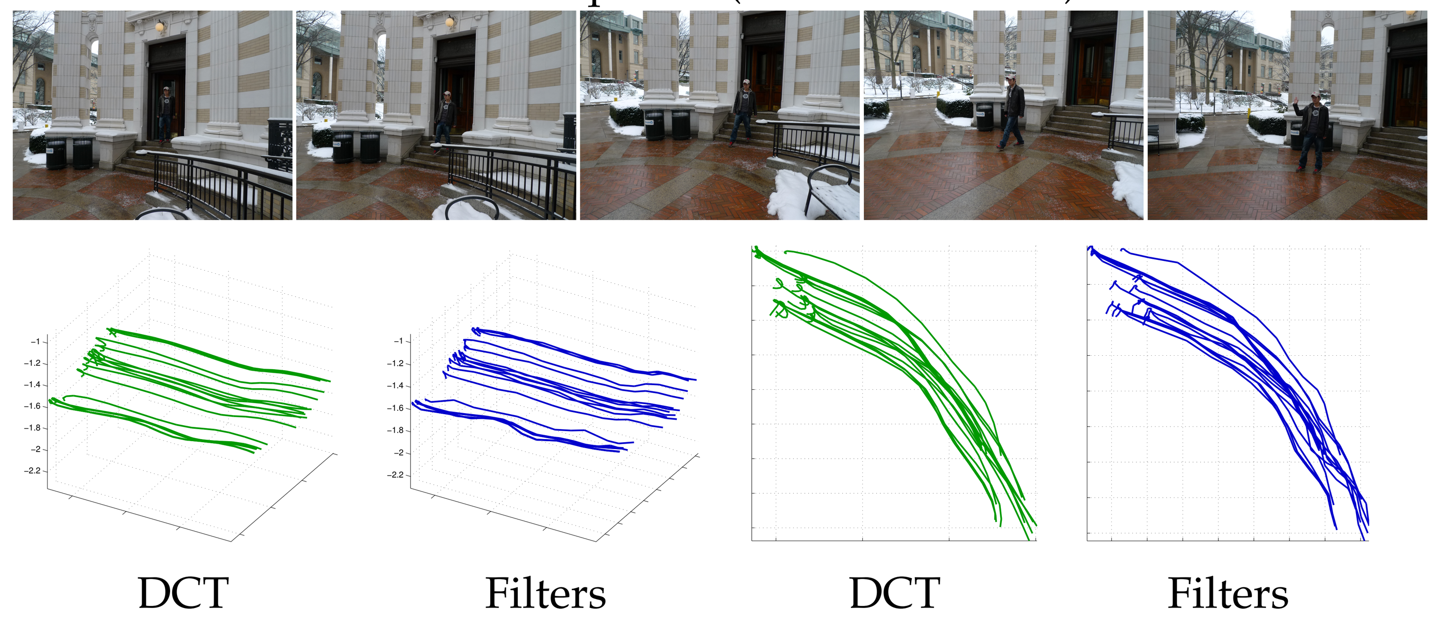


Real Sequences

`dance` sequence (one view shown)



`hand-wave` sequence (one view shown)



References

[1] C. Bregler, A. Hertzmann, and H. Biermann, "Recovering non-rigid 3D shape from image streams," in *CVPR*, 2000.

[2] I. Akhter, Y. Sheikh, S. Khan, and T. Kanade, "Nonrigid structure from motion in trajectory space," in *NIPS*, 2008.

[3] H. S. Park, T. Shiratori, I. Matthews, and Y. Sheikh, "3D reconstruction of a moving point from a series of 2D projections," in *ECCV*, 2010.